Kerr Effect

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Abstract

In this work we present a theoretical treatment of the Kerr effect based on third-order nonlinear optical processes. When an intense electric field is placed in a media, a birefringence is induced and the refractive index depends on the square of the of the amplitude of the electric field. Depending on the nature of the electric field we can describe two phenomena: the electro-optic Kerr effect and the optical Kerr effect. In analogy to these effects we also explore an introduction to the magneto-optical Kerr effect and some different applications.

1 Introduction

Birefringence was first observed in the 17th century when sailors visiting Iceland brought back to Europe calcite crystals that showed double images of objects that were viewed through them. This effect was first described by the Danish scientist Rasmus Bartholin in 1669. However, it was not until 1823 that Augustin-Jean Fresnel described the phenomenon in terms of polarization, understanding light as a wave with field components in transverse polarizations.

In 1875, the Scottish physicist John Kerr observed the change in the refractive index of organic liquids and glasses in the presence of an electric field. This effect is often associated with the birth of nonlinear optics [1]. Kerr found that an isotropic transparent substance becomes birefringent when it is placed in an electric field [2]. The medium takes on the characteristics of an uniaxial crystal whose optic axis corresponds to the direction of the applied field, as shown in Fig(1).

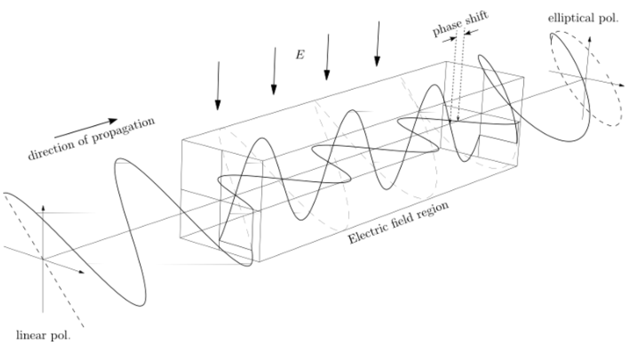

Figure 1: Kerr effect schematically shown. A source of linear polarization propagates in an electric field region. The net result is the appearance of elliptical polarization that indicates the change in the refractive index of the medium.

The quadratic electro-optic effect originates in the birefringence that is induced by the electric field. The invention of the laser in 1960 provided light sources with high-enough electric field strengths to induce the Kerr effect with a second laser that replaces the applied voltage [1]. This latter phenomena is called the Optical Kerr Effect. In this work we are going to discuss three phenomena: the quadratic electro-optical Kerr effect, also called DC Kerr effect (or simply Kerr effect), the optical Kerr effect or AC Kerr effect and and we will explore a brief introduction to the magneto-optical Kerr effect.

2 Intensity-dependent refractive index

Nonlinear Optics is concerned with understanding the behavior of light-matter interactions when the material's response is a nonlinear function of the applied electromagnetic field. The interaction of a beam of light with a nonlinear optical medium can also be described in terms of the nonlinear polarization.

For a nonlinear material, the electric polarization field P will depend on the electric field E:

$$\mathbf{P} = \epsilon_0 \chi^{(1)} \mathbf{E} + \epsilon_0 \chi^{(2)} \mathbf{EE} + \epsilon_0 \chi^{(3)} \mathbf{EEE} + \cdots, \quad (1)$$

where ϵ_0 is the vacuum permittivity and $\chi^{(n)}$ is the n-th order component of the electric susceptibility of the medium. We can write that relationship for the i-th component for the vector $\mathbf{P}$, expressed as:

$$\begin{aligned} P_i = \varepsilon_0 \sum_{j=1}^{3} \chi_{ij}^{(1)} E_j + \varepsilon_0 \sum_{j=1}^{3} \sum_{k=1}^{3} \chi_{ijk}^{(2)} E_j E_k \\ + \varepsilon_0 \sum_{j=1}^{3} \sum_{k=1}^{3} \sum_{l=1}^{3} \chi_{ijkl}^{(3)} E_j E_k E_l + \cdots \end{aligned} \quad (2)$$

For simplicity we are assuming here that the light is linearly polarized and are suppressed the tensor indices of χ; its tensor nature is addressed in the following section. If we consider an electric field with the form $\tilde{E}(t) = E(\omega) \mathrm{e}^{-i\omega t} + c.c.$ The total polarization of the material system is then described by Eq.(3); its derivation and a complete analysis can be found in [3]. However, it is worth pointing out that the different numerical prefactors in Eq.(3) result from the permutation operation in Eq.(2)

$$P^{\mathrm{TOT}}(\omega) = \epsilon_0\chi^{(1)}E(\omega) + 2\epsilon_0\chi^{(2)}E(\omega)E(0) + 3\epsilon_0\chi^{(3)}|E(\omega)|^2E(\omega) + \cdots \tag{3}$$

If we consider the two first terms of Eq.(3), this approximation is known as the linear electro-optic Pockels effect, which leads to an electric-field induced change in the refractive index [4]. On the other side, the part of the nonlinear polarization that influences the propagation of a beam of frequency ω is just given by the third term of Eq.(3), which leads us to the study of the Kerr effect. In this case, we can express the polarization as

$$P(\omega) \cong \epsilon_0\chi^{(1)}E(\omega) + 3\epsilon_0\chi^{(3)}|E(\omega)|^2E(\omega) \equiv \epsilon_0\chi_{\mathrm{eff}}E(\omega), \tag{4}$$

where we have introduced the effective susceptibility $\chi_{\mathrm{eff}} = \chi^{(1)} + 3\epsilon_0\chi^{(3)}|E(\omega)|^2$.

Besides that, the refractive index of many optical materials depends on the intensity of the light used to measure it. The refractive index of many materials can be described by the relation [5]

$$n = n_0 + \bar{n}_2\left\langle \tilde{E}^2 \right\rangle, \tag{5}$$

where n_0 represents the usual zero-field or weak-field refractive index, and $\bar{n}_2$ (sometimes called the second-order index of refraction) gives the rate at which the refractive index increases with the optical intensity. In order to relate the nonlinear susceptibility $\chi^{(3)}$ to the nonlinear refractive index $\bar{n}_2$, we can take $n^2 = 1 + \chi_{\mathrm{eff}}$. Comparing with Eqs.(4) and (5), we can show that the linear and nonlinear refractive indices are related to the linear and nonlinear susceptibilities by

$$n_0 = (1 + \chi^{(1)})^{1/2} \tag{6}$$

$$\bar{n}_2 = \frac{3\chi^{(3)}}{4n_0}. \tag{7}$$

3 Third-order nonlinear processes

The most general third-order nonlinear process involves the interaction of waves at four different frequencies, linked by: $\omega_1 + \omega_2 + \omega_3 = \omega_4$. Fortunately, in all common cases, some of the frequencies are the same, and some may be also zero, or the negatives of the others. The polarization at ω_4 is given by

$$\hat{P}_i(\omega_4) = \frac{1}{2}\epsilon_0 \sum_p \sum_{jkl} \chi^{(3)}_{ijkl}(\omega_4; \omega_1, \omega_2, \omega_3) \hat{E}_j(\omega_1)\hat{E}_k(\omega_2)\hat{E}_l(\omega_3), \tag{8}$$

where $ijkl$ can be x, y or z, and $\sum_p$ indicates the right-hand side is to be summed over all distinct permutations of ω_1, ω_2 and ω_3. This means that the form of the polarization depends on the frequency arranged and it is specific of the kind of process. So, for the *DC Kerr effect* we have $(\omega = 0 + 0 + \omega)$

$$\hat{P}_i(\omega) = 3\epsilon_0 \sum_{jkl} \chi^{K}_{ijkl}(\omega; 0, 0, \omega)\hat{E}_j(0)\hat{E}_k(0)\hat{E}_l(\omega). \tag{9}$$

This concerns refractive index changes caused by an applied DC field.

In the optical or AC Kerr effect, an intense beam of light in a medium can itself provide the modulating electric field, without the need for an external field to be applied. For the *optical Kerr effect* we have $(\omega_1 = \omega_2 - \omega_2 + \omega_1)$

$$\hat{P}_i(\omega_1) = \frac{3}{2}\epsilon_0 \sum_{jkl} \chi^{OK}_{ijkl}(\omega_1; \omega_2, -\omega_2, \omega_1)\hat{E}_j(\omega_2)\hat{E}^*_k(\omega_2)\hat{E}_l(\omega_1). \tag{10}$$

In this case, the refractive index of an optical wave at ω_1 is modified in the presence of a wave at ω_2. It should be also noted that in cases where two (or more) optical waves are involved, these need not necessarily travel in the same direction. There is, for example, no reason in principle why the two waves in Eq.(10) need to be collinear, and in this case their frequencies could even be the same.

3.1 Tensor nature of the third order susceptibility

Let us see how to determine the tensor nature of the third-order susceptibility for the case of an isotropic material such as a glass, a liquid, or a vapor. We begin by considering the general case in which the applied frequencies are arbitrary, and we represent the susceptibility as $\chi_{ijkl} \equiv \chi^{(3)}_{ijkl}(\omega_4 = \omega_1 + \omega_2 + \omega_2)$. In a lossless crystal of the most general triclinic symmetry, there are $3^4 = 81$ independent nonlinear coefficients, for other symmetry classes, the number is lower and a list of independent coefficients in each class can be found in [5]. In an isotropic media, in which all directions are equivalent, the orientation of the $xyz-$ axes can be chosen to make calculations as simple as possible. In this case, only 21 of the 81 coefficients are non-zero, and these are of four types: type 1 (three members) in which all indices are identical ($\chi_1 \equiv \chi_{iiii}$), and types 2, 3 and 4 (six members each) in which two pairs of indices aree the same, namely $\chi_2 = \chi_{jjkk}$, $\chi_3 = \chi_{jkjk}$ and $\chi_4 = \chi_{jkkj}$ $(j \neq k)$.

The numbering scheme ensures that indices 1 and 2 are the same in χ_2, indices 1 and 3 in χ_3, and 1 and 4 in χ_4. Within each type, all members are equal and, as we shall show in a moment, the symmetry of a structurally isotropic medium imposes the further constraint that

$$\chi_1 = \chi_2 + \chi_3 + \chi_4 \tag{11}$$

In terms of indices, the 21 non-zero coefficients can be listed as follows:

$$\begin{aligned}
&1: xxxx = yyyy = zzzz \\
&2: xxyy = yyzz = zzxx = yyxx = zzyy = xxzz \\
&3: xyxy = yzyz = zxzx = yxyx = zyzy = xzxz \\
&4: xyyx = yzzy = zxxz = yxxy = zyyz = xzzx
\end{aligned} \tag{12}$$

The key conclusion is that a structurally isotropic medium has just three independent third-order coefficients. For collinear beams, it makes sense to set the $z-$axis along the direction of propagation, in which case all coefficients involving z in Eq.(12) can be ignored; this reduces the number of relevant non-zero coefficients from 21 to 8. Moreover, if all beams are plane polarized in the same direction, the $x-$axis can be chosen as the direction of polarization, in which case the only relevant coefficient is $\chi_{xxxx} = \chi_1$.

3.2 Electro-optical Kerr effect

In the Electro-optical or DC Kerr effect, a strong DC field changes the refractive index of a medium. If the DC field is y-polarized, Eq.(9) indicates that the respective polarizations in the x and y directions are

$$\begin{aligned}
\hat{P}_x(\omega) &= 3\epsilon_0 \chi^K_{xyyx}(\omega; 0, 0, \omega) E^2_y(0) \hat{E}_x(\omega) \\
&= 3\epsilon_0 \chi^K_4 E^2_y(0) \hat{E}_x(\omega) \\
\hat{P}_y(\omega) &= 3\epsilon_0 \chi^K_{yyyy}(\omega; 0, 0, \omega) E^2_y(0) \hat{E}_y(\omega) \\
&= 3\epsilon_0 \chi^K_1 E^2_y(0) \hat{E}_y(\omega).
\end{aligned} \tag{13}$$

Using Eq.(7), the DC field creates a refractive index difference between the two polarizations given by

$$n_\parallel - n_\perp \cong \frac{3(\chi^K_1 - \chi^K_4) E^2_y(0)}{2n} = \frac{3\chi^K_2 E^2_y(0)}{n}, \tag{14}$$

where $n_\parallel$ and $n_\perp$ are the respective indices for light polarized parallel and perpendicular to the DC field, and n is the zero field refractive index. From Eq.(11), and because χ^K_2 and χ^K_3 are indistinguishable from Eq.(9). The Kerr constant K of a medium is defined by the equation

$$\Delta n \equiv n_\parallel - n_\perp = \lambda_0 K E^2(0), \tag{15}$$

where λ_0 is the wavelength in vacuum and $K = 3\chi^K_2/(\lambda_0 n)$. This difference in index of refraction causes the material to act like a waveplate when light is incident on it in a direction perpendicular to the electric field. If the material is placed between two perpendicular linear polarizers, no light will be transmitted when the electric field is turned off, while nearly all of the light will be transmitted for some optimum value of the electric field.

3.3 Optical Kerr effect

In the optical Kerr effect, a strong wave at frequency ω_2 and intensity $I(\omega_2)$ changes the refractive index of a weak probe wave at ω_1, a process known as cross-phase modulation. If the two waves have the same polarization, the operative term in the polarization is

$$\hat{P}_x(\omega_1) = \frac{3}{2}\epsilon_0 \chi^{OK}_{xxxx}(\omega_1;\ \omega_2, -\omega_2, \omega_1) |\hat{E}_x(\omega_2)|^2 \hat{E}_x(\omega_1), \tag{16}$$

which implies that the refractive index of the weak wave is changed by

$$\Delta n_x \cong \frac{3\chi^{OK}_{xxxx} I(\omega_2)}{2n(\omega_1)n(\omega_2)c\epsilon_0}. \tag{17}$$

If, on the other hand, the weak and strong waves are, respectively, x and y-polarized, Eq.(16) becomes

$$\hat{P}_x(\omega_1) = \frac{3}{2}\epsilon_0 \chi^{OK}_{xyyx}(\omega_1;\ \omega_2, -\omega_2, \omega_1) |\hat{E}_y(\omega_2)|^2 \hat{E}_x(\omega_1). \tag{18}$$

This is the same as Eq.(16) apart from the fact that it contains a type 4 coefficient, and so the index change is weaker. One cannot say that it is three times as weak, because the type 2, 3 and 4 coefficients are not necessarily equal in this case.

An important special case of the optical Kerr effect occurs when a single beam at $\omega = \omega_1 = \omega_2$ modifies its own refractive index. For the case of plane polarized light, Eq.(10) can be written in the simple form

$$\hat{P}_x(\omega) = \frac{3}{4}\epsilon_0 \chi^{OK}_1(\omega;\ \omega, -\omega, \omega) |\hat{E}_x(\omega)|^2 \hat{E}_x(\omega). \tag{19}$$

This implies that the refractive index is changed to

$$n = n_0 + \left(\frac{3\chi^{OK}_1}{4n^2_0 c\epsilon_0} \right) I = n_0 + n_2 I, \tag{20}$$

where I is the intensity, n_0 is the low-intensity index, and the equation defines n_2 as the nonlinear refractive index. It is no surprise that the refractive index change implied by Eq.(20) is essentially the same as that of Eq.(17); the extra factor of 2 in the denominator of n_2 arises from the different pre-factors in Eq.(10).

A full justification of Eqs.(16)-(19) is given in [3] [6]. Again, it is worth pointing out that the different numerical pre-factors in Eqs. (16)-(19) result from the permutation operation in Eq.(8).

4 Applications of the Kerr effect

The optical Kerr effect, plays a significant role in nonlinear optics using high-power pulsed lasers. It is one of the mechanisms contributing to self-focusing in liquids and solids and has also been used as a fast optical shutter for picosecond optical pulses. Some of the areas of application of the Kerr effect includes spectroscopy of liquids including the study of liquid mixtures and the behavior of liquids in nanoconfinement [7], the development of waveguides (devices constructed out of a birefringent material, for which the index of refraction is different for different orientations of light passing through it) [8] and photonic and electro-optic devices [9] [10]. Fig. 2 shows an induced birefringence of graphene oxide liquid crystals with an extremely large Kerr coefficient allowed to fabricate electro-optic devices with macroscopic electrodes, as well as well-aligned, defect-free graphene oxide over wide areas [11].

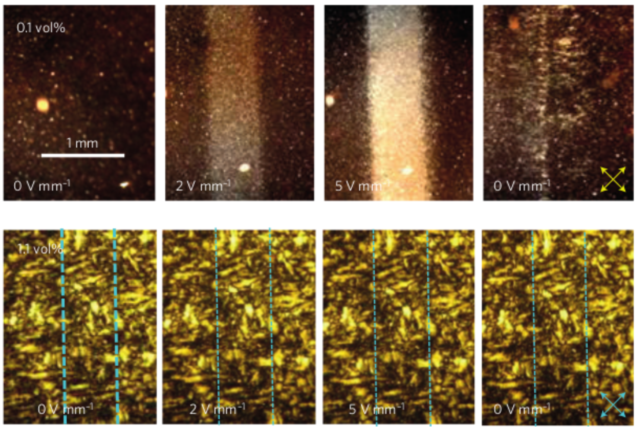

Figure 2: Electric-field-induced birefringence. Top row: Field-induced birefringence was generated by applying electric fields (10 kHz) to an aqueous 0.1 vol% graphene oxide dispersion. When the field was switched off, the field-induced birefringence almost disappeared, with only slight nematic aggregation remaining. Bottom row: In the same cell structure with a 1.1 vol% GO LC, no change was detected up to 20 V mm^{-1} [11].

5 Magneto-optical Kerr effect

The magneto-optic Kerr effect (MOKE) is the phenomenon that the light reflected from a magnetized material has a slightly rotated plane of polarization [12]. It is similar to the Faraday effect where the plane of polarization of the transmitted light is rotated.

Because of the different magnetization directions relative to the plane of the incident light there are three different configurations for MOKE as depicted in Fig.(3). In the polar Kerr effect configuration the magnetization **M** lies perpendicularly to the sample surfaces. In the case of longitudinal Kerr effect **M** lies parallel to the sample surfaces and to the plane of incidence. In the equatorial or transverse configuration **M** lies parallel to the sample surfaces and perpendicular to the plane of incidence.

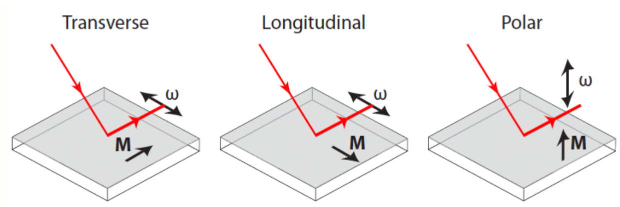

Figure 3: Illustration of variant configurations for the magneto-optic Kerr effect. **M** represents the direction of magnetization. The axis is formed by the plane of incidence indicated by ω [12].

The MOKE has attracted considerable interest in recent years because of its wide application in magneto-optic recording devices. Thanks to its high accuracy, high temporal and spatial resolution and very fast response the MO probe has become a powerful method to study the magnetic properties of ultrathin and multilayer films, and studies of two-dimensional Ising model behavior of ultrathin layers [13] are some out of many examples.

6 Conclusions

A study of some of the different types of electro-optical effects has been presented trying to understand the behavior of the interaction of light with matter when the response of the medium is a non-linear function of the applied electric or magnetic field. It has been explored in some properties of the electric susceptibility tensor, necessary to explain the non-linear effects, considering symmetries in isotropic media. Both the electro-optical and optical Kerr effect and and the magneto-optical have many powerful applications that are already being carried out in research.

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